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Gauge and Yukawa Unification with Broken R–Parity

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Abstract

We study gauge and Yukawa coupling unification in the simplest extension of the Minimal Supersymmetric Standard Model (MSSM) which incorporates R–Parity violation through a bilinear superpotential term. Contrary to what happens in the MSSM, we show that bottom-tau unification at the scale M_{GUT} where the gauge couplings unify can be achieved for any value of $\tan\beta$ by choosing appropriately the sneutrino vacuum expectation value. In addition, we show that bottom-tau-top unification occurs in a slightly wider $\tan\beta$ range than in the MSSM.

The Standard Model (SM) of particle physics is very successful in describing the interactions of the elementary particles, except possibly neutrinos. Although it is regarded as a good low-energy effective theory, the SM has many unanswered questions and theoretical problems. Its gauge symmetry group is the direct product of three groups $SU(3) \times SU(2) \times U(1)$ and the corresponding gauge couplings are unrelated. It does not explain the three family structure of quarks and leptons, and their masses are fixed by arbitrary Yukawa couplings, with neutrinos being prevented from having mass. The Higgs sector, responsible for the electroweak symmetry breaking and for the fermion masses, has not been verified experimentally and the Higgs mass is unstable under radiative corrections. As a result, say, the hierarchy between the electroweak scale and the Planck scale is not understood.

In supersymmetry (SUSY) [1] the Higgs mass is stabilized under radiative corrections because the loops containing standard particles is partially cancelled by the contributions from loops containing supersymmetric particles. If we add to the Minimal Supersymmetric Standard Model (MSSM) [2] the notion of Grand Unified Theory (GUT), then we find that the three gauge couplings unify at a certain scale M_{GUT} [3]. Indeed, measurements of the gauge couplings at the CERN e^+e^- collider LEP and neutral current data [4] are in good agreement with the MSSM–GUT with the SUSY scale $M_{SUSY} \lesssim 1$ TeV [5]. In addition, the unification scale in SUSY–GUT is high enough to predict a proton decay rate slower than present experimental limits, as opposed to the non–SUSY GUTs, where the proton decays too fast.

Besides achieving gauge coupling unification [6], GUT theories also reduce the number of free parameters in the Yukawa sector. For example, in $SU(5)$ models, the bottom quark and the tau lepton Yukawa couplings are equal at the unification scale, and the predicted ratio m_b/m_τ at the weak scale agrees with experiments. Furthermore, a relation between the top quark mass and $\tan\beta$, the ratio between the vacuum expectation values of the two Higgs doublets is predicted. Two solutions are possible, characterized by low and high values of $\tan\beta$ [7]. In models with larger groups, such as $SO(10)$ and E_6, both the top and bottom Yukawa couplings are unified with the tau Yukawa at the unification scale [8]. In this case, only the large $\tan\beta$ solution survives.

Recent global fits of low energy data to minimal supersymmetry [10] show that it is hard to reconcile these constraints with the large $\tan\beta$ solution. Specially important are the measurements of the $B(b \to s\gamma)$ decay rate and the bound on the lightest Higgs mass. In addition, the low $\tan\beta$ solution with $\mu < 0$ is also disfavoured. In this letter, we show that the minimal extension of the MSSM–GUT [11] in which R–Parity violation is introduced via a bilinear term in the MSSM superpotential [12, 13], allows $b - \tau$ Yukawa unification for any value of $\tan\beta = v_u/v_d$ and satisfying perturbativity of the couplings. We also analize the $t - b - \tau$ Yukawa unification and find that it is easier to achieve than in the MSSM, occurring in a slightly wider high $\tan\beta$ region.

For simplicity, we consider only the third generation of quarks and leptons. In this way, the superpotential is given by

$$W = h_t \widehat{Q}_3 \widehat{U}_3 \widehat{H}_u + h_b \widehat{Q}_3 \widehat{D}_3 \widehat{H}_d + h_\tau \widehat{L}_3 \widehat{R}_3 \widehat{H}_d + \mu \widehat{H}_u \widehat{H}_d + \epsilon_3 \widehat{L}_3 \widehat{H}_u \tag{1}$$

where the first four terms correspond to the MSSM and the last one is the bilinear term which violates R–parity. This superpotential is motivated by models of spontaneous breaking of R–Parity [14]. Here, R–Parity and lepton number are violated explicitly by the ϵ_3 term.

It is clear from eq. (1) that the scalar potential contains terms which induce a non–zero vacuum expectation value (VEV) of the tau sneutrino $\langle \tilde{\nu}_\tau \rangle = v_3/\sqrt{2}$. It contributes to the W mass according to $m_W^2 = \frac{1}{4} g^2 (v_d^2 + v_u^2 + v_3^2)$, where $v_d/\sqrt{2}$ and $v_u/\sqrt{2}$ are the VEVs of the two Higgs doublets H_d and H_u respectively. The R–Parity violating parameters ϵ_3 and v_3 violate tau–lepton number, inducing a non-zero ν_τ mass $m_{\nu_\tau} \propto (\mu v_3 + \epsilon_3 v_d)^2$, which arises due to mixing between the weak eigenstate ν_τ and the neutralinos. The latest ν_τ mass limit from ALEPH is $m_{\nu_\tau} \lesssim 16$ MeV. The ν_e and ν_μ remain massless in first approximation. They acquire typically smaller masses from supersymmetric loops. As already mentioned, in what follows we consider only the third generation of quarks and leptons.

It is important to note that the ϵ–term in eq. (1) is a physical parameter and cannot be eliminated by a redefinition of the superfields $\widehat{H}_d$ and $\widehat{L}_3$ [15]. The reason is that, after the rotation, bilinear terms which induce a tau sneutrino VEV are re–introduced in the soft scalar sector [16]. Moreover, in contrast to many prejudices [17], we wish to stress that the R–Parity violating parameters v_3 and ϵ_3 need not be small. In models with universality of soft supersymmetry breaking mass parameters [16] m_{ν_τ} is naturally small because it arises from a seesaw mechanism in which the the *effective* mixing arises only radiatively, and can easily lie in the eV range [11].

R–Parity violation also implies that the charginos mix with the tau lepton, through a mass matrix is given by

$$\boldsymbol{M}_C = \begin{bmatrix} M & \frac{1}{\sqrt{2}} g v_u & 0 \\ \frac{1}{\sqrt{2}} g v_d & \mu & -\frac{1}{\sqrt{2}} h_\tau v_3 \\ \frac{1}{\sqrt{2}} g v_3 & -\epsilon_3 & \frac{1}{\sqrt{2}} h_\tau v_d \end{bmatrix} \tag{2}$$

with h_τ being the tau Yukawa coupling. Imposing that one of the eigenvalues reproduces the observed tau mass m_τ, the tau Yukawa coupling can be solved exactly as [13]

$$h_\tau^2 = \frac{2 m_\tau^2}{v_d} \left[\frac{1 + \delta_1}{1 + \delta_2} \right] \tag{3}$$

where the δ_i , $i = 1, 2$, depend on m_τ, on the SUSY parameters $M, \mu, \tan\beta$ and on the R-parity violating parameters ϵ_3 and v_3. They can be found in ref. [13] and can easily be shown to vanish in the MSSM limit $\epsilon_3 \to 0$ and $v_3 \to 0$. On the other hand, the bottom and top Yukawa couplings are related to the bottom and top masses according to

$$m_t = h_t \frac{v}{\sqrt{2}} \sin\beta \sin\theta \,, \quad m_b = h_b \frac{v}{\sqrt{2}} \cos\beta \sin\theta \tag{4}$$

where we use spherical coordinates for the VEVs, defining $v = 2m_W/g$, $\tan\beta = v_u/v_d$, and $\cos\theta = v_3/v$.

We now turn to the study of the renormalization group evolution of the various relevant parameters of the model such as the gauge and Yukawa couplings, the SM quartic Higgs coupling and the third generation fermion masses. In our approach we divide the evolution into three ranges: (i) from M_{SUSY} to M_{GUT}, where we use the two-loop RGEs of our model, (ii) from m_t to M_{SUSY}, where we use the two-loop SM RGEs including the quartic Higgs coupling and (iii) from M_Z to m_t we use running fermion masses and gauge couplings.

Using a top-bottom approach we randomly vary the unification scale M_{GUT} and the unified coupling α_{GUT} looking for solutions compatible with the low energy data [18] $\alpha_{em}^{-1}(m_Z) = 128.896 \pm 0.090$, $\sin^2\theta_w(m_Z) = 0.2322 \pm 0.0010$, and $\alpha_s(m_Z) = 0.118 \pm 0.003$. We use the approximation of a common decoupling scale $M_{SUSY} \lesssim 1$ TeV for all the supersymmetric particles. The solutions we find are concentrated in a region of the $M_{GUT} - \alpha_{GUT}$ plane. For the simpler case where the SUSY scale coincides with the top mass, $M_{SUSY} = m_t$, this region is centered at the point $M_{GUT} \approx 2.3 \times 10^{16}$ GeV and ${\alpha_{GUT}}^{-1} \approx 24.5$, which we adopt from now on.

Next, we study the unification of Yukawa couplings using two-loop RGEs. We take $m_W = 80.41 \pm 0.09$ GeV, $m_\tau = 1777.0 \pm 0.3$ MeV, and $m_b(m_b) = 4.1$ to 4.5 GeV [18]. We calculate the running masses $m_\tau(m_t) = \eta_\tau^{-1} m_\tau(m_\tau)$ and $m_b(m_t) = \eta_b^{-1} m_b(m_b)$, where η_τ and η_b include three–loop order QCD and one–loop order QED [9]. At the scale $Q = m_t$ we keep as a free parameter the running top quark mass $m_t(m_t)$ and vary randomly the SM quartic Higgs coupling λ. Using SM RGEs we evolve the gauge, Yukawa, and Higgs couplings from $Q = m_t$ up to $Q = M_{SUSY}$. The initial conditions for the SM Yukawa couplings are $\lambda_i^2(m_t) = 2m_i^2(m_t)/v^2$, with $i = t, b, \tau$ and $v = 246.2$ GeV.

At the scale $Q = M_{SUSY}$, below which all SUSY particles are decoupled (including the heavy Higgs bosons) we impose the following boundary conditions for the quark Yukawa couplings

$$\begin{aligned} \lambda_t(M_{SUSY}^-) &= h_t(M_{SUSY}^+) \sin\beta \sin\theta \\ \lambda_b(M_{SUSY}^-) &= h_b(M_{SUSY}^+) \cos\beta \sin\theta \end{aligned} \tag{5}$$

where h_i denote the Yukawa couplings of our model and λ_i those of the SM. Due to its mixing with charginos, the boundary condition for the tau Yukawa coupling is slightly

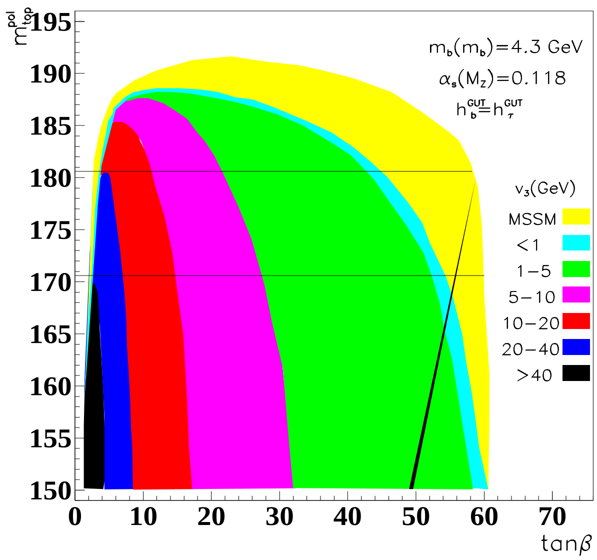

Figure 1: Top quark mass as a function of $\tan\beta$ for different values of the R–Parity violating parameter v_3. Bottom quark and tau lepton Yukawa couplings are unified at M_{GUT}. The horizontal lines correspond to the 1σ experimental m_t determination. Points with $t - b - \tau$ unification lie in the diagonal band at high $\tan\beta$ values. We have taken $M_{SUSY} = m_t$.

more complicated:

$$\lambda_\tau(M_{SUSY}^-) = h_\tau(M_{SUSY}^+)\cos\beta\sin\theta\sqrt{\frac{1+\delta_2}{1+\delta_1}} \tag{6}$$

Finally, the boundary condition for the quartic Higgs coupling is given by

$$\lambda(M_{SUSY}^-) = \frac{1}{4}\Big[(g^2(M_{SUSY}^+) + g'^2(M_{SUSY}^+)\Big](\cos 2\beta\sin^2\theta + \cos^2\theta)^2 \tag{7}$$

The MSSM limit is obtained setting $\theta \to \pi/2$ i.e. $v_3 = 0$.

At the scale $Q = M_{SUSY}$ we vary randomly the SUSY parameters M, μ and $\tan\beta$, as well as the R–Parity violating parameter ϵ_3. The parameter $v_3 = v\cos\theta$ is calculated from eq. (7). Since λ (or equivalently the SM Higgs mass $m_H^2 = 2\lambda v^2$) is varied randomly, in practice we also scan over θ. This way, we consider all possible initial conditions for the RGEs at $Q = M_{SUSY}$, and evolve them up to the unification scale $Q = M_{GUT}$. The solutions that satisfy $b - \tau$ unification are kept.

In Fig. 1 we illustrate our point by plotting the top quark mass (we always use the pole mass) as a function of $\tan\beta$. For simplicity we have taken $M_{SUSY} = m_t$ but it should be clear that a different M_{SUSY} choice would not change qualitatively our results. Each selected point in our scan satisfies bottom-tau unification (to within a 1%) $h_b(M_{GUT}) = h_\tau(M_{GUT})$ and it is placed in one of the shaded regions according to the value of $|v_3|$. The first region with $v_3 = \epsilon_3 = 0$ corresponds to the MSSM and sits at the top of the plot. Points with $|v_3| < 1$ GeV fall in the region just below. The subsequent regions labelled by $1 < |v_3| < 5$ GeV up to $|v_3| > 40$ GeV respectively are obtained when v_3 gets higher. They are narrower in $\tan\beta$. Note that points with smaller v_3 values, say $1 < |v_3| < 5$ GeV fall in the region labelled as such, as well as in all previous regions, but not in the subsequent ones. This overlapping with the previous regions decreases as we increase $|v_3|$ in such a way that points with $|v_3| > 40$ GeV fall almost exclusively in the last region.

The two horizontal lines correspond to the top quark mass within a 1σ error. In the MSSM limit we can see the two solutions compatible with the experimental value of the top quark mass, one with $\tan\beta \approx 1$ and the other with $\tan\beta \approx$ 55-60. It is clear from the figure that by selecting appropriately the value of $|v_3|$ we can find $b-\tau$ unification for any $\tan\beta$ value within the perturbative region $1 \lesssim \tan\beta \lesssim 62$ of the Yukawa couplings. For $|v_3| \lesssim 20$ GeV one has, as in the MSSM, two disconnected solutions for $b-\tau$ unification, one with $\tan\beta \approx 1$, and a large $\tan\beta$ range which, for intermediate v_3 can be quite broad. Note that for $20 < |v_3| < 40$ GeV only the $\tan\beta$ range from 3 to 8 or so is consistent with the 1 σ top mass measurement, for the chosen α_s and $m_b(m_b)$ values. Similarly, the $|v_3|$ range above 40 GeV would be ruled out. Note that our results do not depend qualitatively on the definition chosen for $\tan\beta$. For example, if we define $\tan\beta$ in the way which is natural in the basis where the ϵ_3–term disappears from the superpotential, $\tan\beta' \equiv v_u/\sqrt{v_d^2 + v_3^2}$ we also can find $b-\tau$ unification for any $\tan\beta'$ value.

We now turn to the discussion of the uncertainties of the unification program in this model. The general trend follows closely that of the MSSM. The dependence of our results on the strong coupling constant and the bottom mass running is totally analogous to what happens in the MSSM. Indeed, we have studied the effect of varying α_s in Fig. 1 and found that the upper bound on $\tan\beta$, which is $\tan\beta \lesssim 61$ for $\alpha_s = 0.118$, increases with α_s and becomes $\tan\beta \lesssim 63$ (59) for $\alpha_s = 0.122$ (0.114). On the other hand the MSSM region is narrower if α_s increases, specially at high $\tan\beta$ values. We have verified that the same trend extends to the regions with large v_3. Finally, we mention that the top mass value for which unification is achieved for any $\tan\beta$ value within the perturbative region increases with α_s, as in the MSSM. Turning to the dependence on m_b, the behaviour is the opposite one. In Fig. 1 we have taken $m_b(m_b) = 4.3$ GeV. As before the value of $\tan\beta$ is bounded from above by $\tan\beta \lesssim 61$ due to the perturbativity condition of the bottom quark Yukawa coupling. If we consider $m_b(m_b) = 4.1$ (4.5) GeV then the upper bound of this parameter is given by $\tan\beta \lesssim 64$ (58). In addition, the MSSM region is narrower (wider) at high $\tan\beta$ compared with the $m_b(m_b) = 4.3$ GeV case shown in Fig. 1.

Finally we have studied the possibility of bottom-tau-top unification in our model. The diagonal line at high $\tan\beta$ values corresponds to points where $t-b-\tau$ unification is achieved. Since the region with $|v_3| < 5$ GeV overlaps with the MSSM region, it follows that $t-b-\tau$ unification is possible in this model for values of $|v_3|$ up to about 5 GeV, instead of 50 GeV or so, which holds in the case of bottom-tau unification. Within the MSSM, t–b–τ unification is achieved in the range $56 \lesssim \tan\beta \lesssim 59$ with m_t completely inside the 1σ region. In this case, bilinear R–Parity violation does not enlarge the allowed $\tan\beta$ region. However, at the 2σ level our model allows $t-b-\tau$ unification for $54 \lesssim \tan\beta \lesssim 59$. In addition, we have checked that the region with $t-b-\tau$ unification in the MSSM case shrinks if α_s is increased. The space left out by the MSSM is taken over by the regions with $|v_3| < 5$ GeV so that, for large α_s, $t-b-\tau$ unification occurs in a wider $\tan\beta$ range than possible in the MSSM, even in the 1σ level.

In conclusion, we have summarized [19] the results of the first systematic study of gauge and Yukawa coupling unification in a model where we introduce bilinear R–Parity violation. The model is the simplest alternative to the MSSM which mimics in an effective way many of the features of models of spontaneous breaking of R–Parity. We showed that, in contrast to the MSSM, where bottom-tau unification is achieved in two disconnected $\tan\beta$ regions, in our model $b-\tau$ unification occurs for any $\tan\beta$ value, provided we choose appropriately the value of the tau sneutrino vacuum expectation value v_3. In addition, we showed that $t-b-\tau$ unification is achieved for $|v_3| \lesssim 5$ GeV at high values of $\tan\beta$ in a slightly wider region than that of the MSSM. Apart from the intrinsic interest in the study of broken R–parity models, because of their theoretical as well as phenomenological importance, our results are relevant in connection with LEP bounds on the Higgs boson mass and recent measurements of the $B(b \to s\gamma)$ decay rate. Taken at face value, these disfavour the high $\tan\beta$ solution and also the low $\tan\beta$ solution with $\mu < 0$ in the MSSM as suggested in [10].

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